Effect of Shear Stresses in Indirect Compression Tests of Unidirectional Carbon Fiber/Epoxy

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The effect of the presence of shear stresses on the compressive strength of unidirectional carbon fiber/epoxy has been analyzed using Rosen's model with experimental data for the nonlinear stress-strain response of the epoxy resin. It is shown that shear stresses of the order of 50 MPa are sufficient to reduce the compressive strength from the very high values predicted by Rosen's equation to the levels normally measured. Finite element analysis has been carried out on two standard indirect compression specimens, including the effect of nonlinear shear behavior of the composite. Shear stresses well over 50 MPa were found in both the ASTM and CRAG short gauge length specimens near the end of the tabs where failure usually occurs. It is suggested that these tests underestimate the true ultimate compressive strength of carbon fiber/epoxy by a significant margin due to the presence of shear stresses. This is supported by high strains to failure reported for compression of laminated plates, bending of unidirectional carbon fiber/epoxy, and compression of single-fiber bundles embedded in epoxy.

I. Introduction

IT is well known that design allowables for the ultimate compressive strength of carbon fiber composites are much lower than for ultimate tensile strength. Typical values for unidirectional high-strength carbon fiber/epoxy are 1300 MPa in compression compared with 2000 MPa in tension. The design of carbon fiber structures is often controlled by compressive strength, and so this is a major contributor to the limitation of performance benefits realized from carbon fiber composites.

The most common method of measuring the compressive strength of unidirectional composites is by indirect compression tests. In these tests the specimens are not loaded directly in compression but rather the compression load is applied indirectly by shear through tabs at the ends of the specimen. However, when compressive strength values obtained from indirect compression tests are used in conjunction with classical laminated plate theory to predict the compressive strength of laminates, the strength is often underestimated. For example, Kretsis et al.[1] report tensile and compressive tests on 0/90/±45-deg laminates of 60% XAS/913. Results for three different layups were compared with predictions from laminated plate theory using the Tsai-Hill failure criterion. In tension, good agreement was obtained, with the measured strengths exceeding the predictions by between 1.3 and 6.5%. However, in compression the measured strengths exceeded the predictions by between 15.5 and 19.7%. The corresponding strains to failure were between 22.9 and 23.6% higher than expected. It was concluded that the strengths were underestimated because the predictions were based on unidirectional compressive strengths that may have been too low as a result of premature failure in a combination of compression and shear instability. While these results are conservative and may be acceptable for initial design purposes, they do indicate that the material may be significantly stronger in compression than generally believed.

There is usually considerable scatter in compressive strength measurements, and in some cases very high values have been reported. Woolstencroft et al.[2] reported tests on 60% volume fraction XAS/914 using a variety of methods. They reported a mean value of compressive strength of 1400 MPa. However, significantly higher strengths were found with the modified Celanese specimen, with a mean value of 1840 MPa and one specimen failing at 1990 MPa. These results were attributed to the effect of transverse compressive stresses in the specimen. However, the fact that it was possible to obtain such high values points to the material having a higher intrinsic strength than normally attributed to it.

There are several other experiments in which very high compressive strains have been obtained. For example, work at Bristol University on 60% volume fraction XAS/913 specimens subject to four-point bending has shown that failure normally occurs on the tension side, despite the large stress concentration at the rollers on the compression side. Assuming linear elastic behavior, this implies that the compressive strength is higher than the tensile strength. Strains of over 2% have been measured on the compression side of bending specimens with no apparent damage. This compares with a failure strain of the order of 1% when tested in nominally pure compression using the IITRI test fixture and CRAG specimen. In fact, carbon fiber/epoxy shows significant nonlinearity at high strains. Taking this into account, 2% strain is believed to correspond to a stress of about 1900 MPa.[3] This is considerably in excess of measured compressive strengths. Similar results with compressive strains over 2% in bending tests with XAS/913 have been reported elsewhere.[4] In tests at Bristol University on XAS/913 specimens subject to combined bending and compression, compressive strains of over 2.5% were obtained with no apparent damage.

Tests on single bundles of a variety of PAN-based carbon fibers embedded in epoxy resin have shown compressive strains to failure of between 2.11 and 2.65%.[5,6] Similar failure modes of microbuckling and kinking were observed as in ordinary high volume fraction material. These values are considerably higher than corresponding fiber tensile strains to failure.

All of these results point toward the strength and strain to failure of unidirectional carbon fiber/epoxy being significantly in excess of the values normally measured by indirect compression tests. The true compressive strength and strain to failure may actually approach or even exceed the values for tension.

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The effect of the presence of shear stresses in this type of test does not appear to have received very much attention. In the present paper, it is shown on the basis of Rosen's model that the presence of shear stresses can be expected to significantly reduce the compressive strength of unidirectional carbon fiber/epoxy. It is also shown by finite element analysis that large shear stresses are present in indirect compression specimens. It is therefore suggested that indirect compression tests underestimate compressive strength by a significant margin due to the presence of these shear stresses.

II. Effect of Shear Stresses on Compressive Strength

A number of theories for compressive strength of unidirectional composites have been proposed and these have been reviewed by Shuart.[7] The classic study of Rosen[8] gave an equation for the compressive strength σ_c of the form:

$$\sigma_c = \frac{G_m}{1 - V_f} \tag{1}$$

where G_m is the shear modulus of the matrix and V_f the fiber volume fraction. This is based on buckling of the fibers in phase due to insufficient matrix shear stiffness, which is generally agreed to be the dominant failure mode in current high strength carbon fiber/epoxy composites. It is well known that Eq. (1) overestimates the compressive strength. Most of the subsequent developments have been concerned with the effect of initial curvature in the fibers, which it has been shown can greatly reduce the strength.[9,10] It has also been pointed out that very small misalignments between the fibers and the loading axis drastically reduce the theoretical compressive strength.[11] The misalignments induce shear stresses, which reduce the matrix tangent shear modulus and hence its ability to support the fibers against buckling. A similar effect can arise due to the presence of shear stresses in compressive specimens. There are significant shear stresses in indirect compression tests that lead to a reduction in the effective matrix tangent shear modulus. This would be expected to significantly reduce the compressive strength.

The effect of shear stresses on the compressive strength of a unidirectional composite can be shown using Rosen's Eq. (1) with nonlinear matrix shear properties. It is assumed that the presence of shear stresses does not alter the failure mode, which is still due to shear instability in the matrix. However, the shear stresses are important because they result in a reduction in the matrix shear modulus. The matrix properties for the work presented here are based on results reported in Ref. 12. Small blocks of 15 × 15 × 8 mm neat 914C cured resin were tested in compression between two platens. PTFE sheets were placed between the resin and the end platens to minimize the effect of friction. Strain gauges were used to measure the longitudinal and transverse strains. The width and thickness were measured during the test so that the true stress based on the true area could be calculated. The shear stress-strain curve was derived from the compression stress-strain and Poisson ratio data assuming the relation for isotropic behavior:

$$G = E/2\,(1 + \nu) \tag{2}$$

where G, E, and ν are the shear modulus, Young's modulus, and Poisson's ratio, respectively. The shear stress-strain curve for the resin is shown in Fig. 1. Expressions for shear modulus as a function of shear strain derived by curve fitting are presented in Appendix A. In order to assess the effect of shear stresses on compressive strength, values of the matrix tangent modulus under given applied shear stresses are required. Since the expressions for the modulus given in Appendix A are in terms of shear strain, a simple iterative procedure is necessary to derive the shear strain and hence the tangent modulus at each value of applied shear stress. Using these calculated tangent moduli in Eq. (1) with a fiber volume fraction of 60% gives the curve of predicted compressive strength against applied shear stress shown in Fig. 2.

It can be seen that there is a steady reduction in predicted compressive strength with increasing shear stress. The strength under pure compression is 4100 MPa, which as expected is far higher than values normally measured. Shear stresses in the range 40–55 MPa give compressive strengths in the range 1000–1600 MPa, which is the normal range of measured values. At higher shear stresses the strength falls further, eventually going to almost zero at about 70 MPa shear stress at which point the matrix has completely yielded.

There may be a further reduction in the composite compressive strength due to interaction between compressive stresses and shear stresses in the matrix, although this effect has not been included in the simple analysis presented here. Under the combination of compressive and shear stresses the matrix might be expected to yield more easily, giving rise to a reduction in shear tangent modulus and hence a reduction in predicted compressive strength.

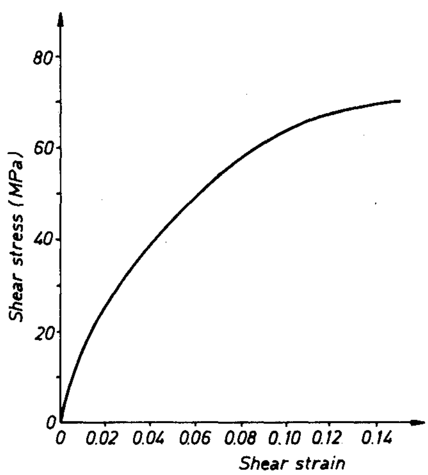

Fig. 1 Shear properties for neat 914C resin.

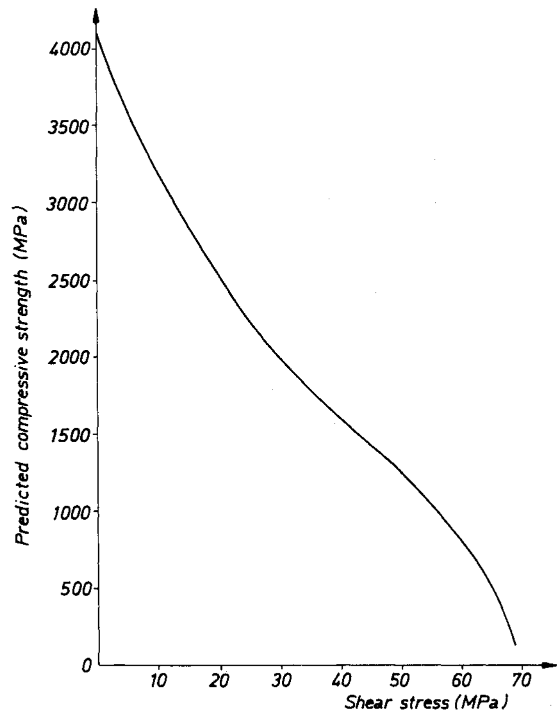

Fig. 2 Effect of shear stresses on the predicted compressive strength of 0-deg XAS/914.

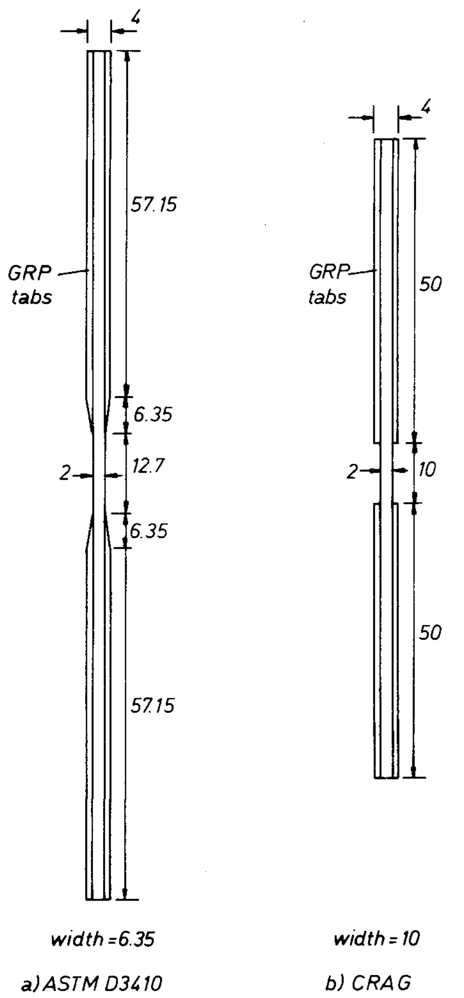

Fig. 3 Geometry of indirect compression specimens (dimensions in mm).

III. Finite Element Analysis of Compression Test Specimens

Indirect compression tests on unidirectional carbon fiber/epoxy are carried out using short, straight-sided specimens with tabs through which the load is introduced. These tabs may be tapered as in the case of the ASTM D3410 specification[13] or straight as in the CRAG specification.[14] Specimens are tested in a fixture such as the Celanese or IITRI rig.[13] Significant shear stresses are introduced into the specimen since the load is deliberately applied by shear. To investigate the magnitude of these shear stresses, finite element analysis was undertaken of both these specimen configurations.

The geometry of the specimens is shown in Fig. 3. A three-dimensional analysis is required to fully analyze the stresses, but since the main interest is in the shear stresses arising in the longitudinal plane normal to the plane of the specimen, a two-dimensional analysis was considered satisfactory. The section of specimen gripped in the fixture is subject to a state of approximately plane strain, whereas the central section is subject to nominally plane stress. For the ASTM specimen, analyses assuming both plane strain and plane stress for the entire specimen were carried out. Very little difference was found in the shear stresses for these two cases. Plane stress was assumed for subsequent analyses.

The ABAQUS finite element program was used for the analysis.[15] Because of symmetry, only one quarter of the specimen had to be modeled. Elements of type CPS8R and CPS6 were used. These are eight- and six-noded continuum plane stress elements with parabolic interpolation of displacement. The eight-noded elements use reduced integration. Small element sizes were used in the areas of expected high stress gradients and much larger ones away from these areas. Five elements were used through the half thickness of the specimen and five through the tab. The number of elements through the thickness was kept constant, and because of this and the geometry of the specimen, some elements had very high aspect ratios. However, the aspect ratios in the areas of interest were considered acceptable. The finite element meshes used are shown in Fig. 4.

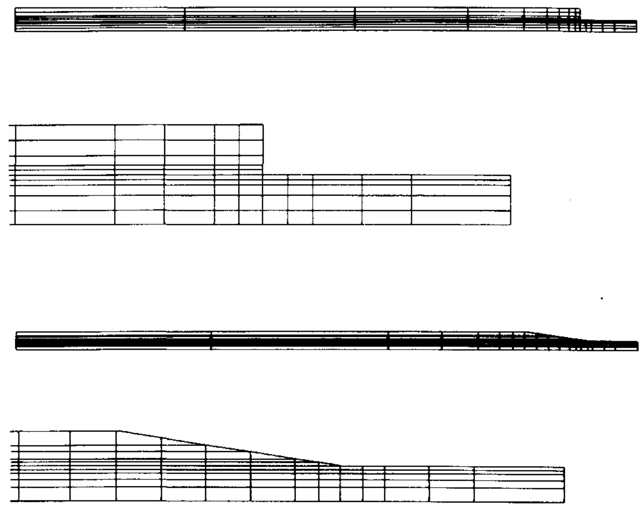

Fig. 4 Finite element meshes for analysis of compression tests.

The loading fixtures are very stiff in comparison with the tabs. All nodes on the gripped portion of the tabs were therefore constrained to have equal displacements in the loading direction and also in the through thickness direction. Since the nodes on the tab surface were constrained together, the load could be applied as a single point force to any of these nodes. As well as the applied shear load, the compressive load through the thickness of the specimen due to the tapered sleeves on the fixture was applied. The magnitude of the compression load F_c can be calculated in terms of the applied shear load P by considering equilibrium of the sleeves. This gives an equation

$$F_c = P \; \frac{\cos\theta - \mu \sin\theta}{\sin\theta + \mu \cos\theta} \tag{3}$$

where θ is the taper angle and μ the coefficient of friction between the two parts of the fixture. Assuming the usual taper angle of 10 deg, and a coefficient of friction of 0.3, the compressive load is 1.988 times the shear load. The compressive strength of XAS/914 is considered to be about 1400 MPa.[2] A load sufficient to cause an average longitudinal stress of this magnitude was therefore applied to each specimen. Symmetric boundary conditions were applied to all nodes on the midplane of the specimen and on the center plane normal to the loading direction.

Material properties were based on data from Ref. 16. The longitudinal and through thickness moduli and Poisson's ratio were assumed not to be functions of strain. In practice there is some nonlinearity of response, but since the main objective was to investigate the magnitude of the shear stresses, it was considered acceptable to neglect this. The through thickness modulus was based on transverse compression tests. Nonlinear shear stress-strain behavior was included using in-plane data derived from tension tests on ±45-deg laminates. The shear behavior was assumed to be elastic rather than elastic-plastic. However, since only monotonically increasing loading is considered, plastic flow in the matrix will not affect the shear stresses. No interaction of stress components was considered and so the shear response was assumed to be indepen-

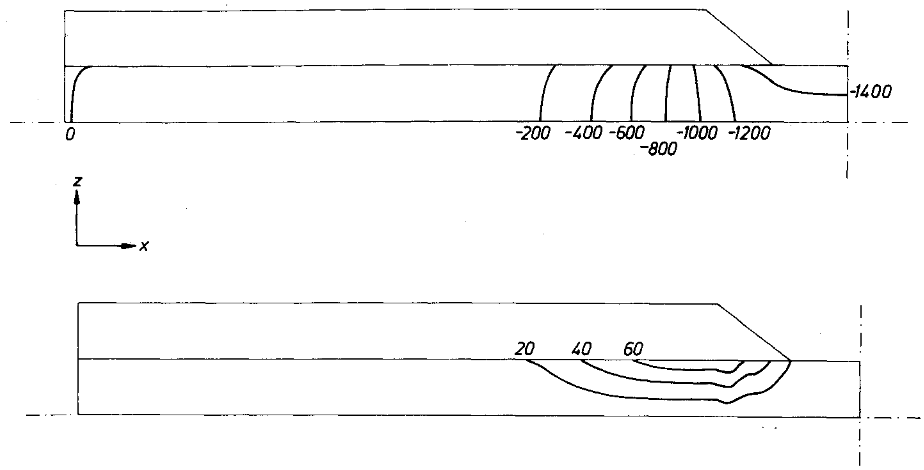

Fig. 5 Stress contours for ASTM D3410 compression specimen: a) Compressive stress σ_x (MPa); b) Shear stress τ_{xz} (MPa).

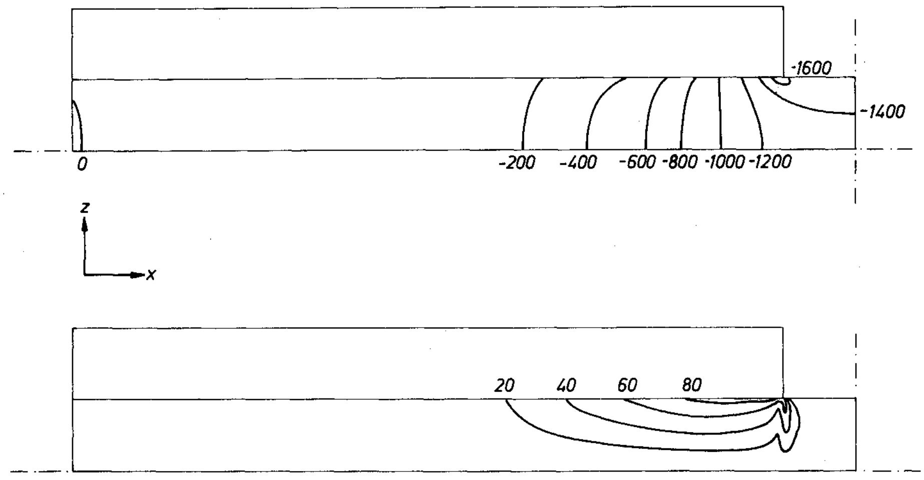

Fig. 6 Stress contours for CRAG compression specimen: a) Compressive stress σ_x (MPa); b) Shear stress τ_{xz} (MPa).

dent of the magnitude of direct stresses or strains. A user subroutine was written to represent this material behavior in the ABAQUS program. End tabs were assumed to be made from cross-plied glass fiber-epoxy. The CRAG specification recommends glass fiber or light alloy tabs. The ASTM specification recommends steel tabs for unidirectional carbon fiber-epoxy; however, glass fiber tabs are more commonly used. The glass fiber was assumed to be transversely isotropic, with linear elastic direct stress-strain and nonlinear shear stress-strain characteristics. Material data used is given in Appendix B.

Results in the form of contour plots of longitudinal and shear stresses are shown in Figs. 5 and 6. To make the plots clearer, the thickness dimensions have been scaled up by a factor of 5.

The CRAG specimen shows a uniform longitudinal stress distribution across the central section, with the stress varying from -1390 MPa in the center to -1408 MPa at the surface. The stress rises to over -1600 MPa at the stress concentration at the end of the tab. A peak value of over -2000 MPa was obtained at the corner node, however this value should be disregarded since there is a discontinuity here and the exact value of stress obtained will depend on the mesh size. Stresses fall steadily, going further into the tabbed part of the specimen, with quite a uniform distribution across the section. The end half of the length of specimen modeled is virtually unstressed. There is a zone of very high shear stresses extending back about 5 mm from the end of the tabs. Stresses are over 80 MPa at the joint between the tabs and the specimen with a peak value of 90 MPa. Stresses fall steadily to zero in the center. Shear stresses fall steadily, going further along the tab to almost zero halfway along the tab. The zone of high shear stresses does extend slightly forward of the end of the tab in the interior of the specimen, although the shear stresses here are lower, of the order of 40 MPa. The fluctuation of shear stress that occurs immediately in line with the end of the tab is due to the discontinuity caused by the tab. There is a very severe stress gradient, especially at the surface where the shear stress has to go from a high value at the end corner of the tab to zero over one element. This causes a perturbation of the stress field in the immediate vicinity of the end of the tab but should not affect the results away from this area. There are also significant compressive stresses in the through thickness direction, but these have not been shown since they are of less interest in the present context.

For the ASTM specimen, the stress distributions are quite similar. The longitudinal stress is uniform across the central section. The stress concentration at the end of the tabs is much less severe, with a peak value of around -1600 MPa. Shear stresses are slightly lower than in the CRAG specimen. Peak values are about 75 MPa close to the start of the tapered section of the tabs.

IV. Discussion

It has been shown that shear stresses have a significant effect on the theoretical compressive strength of unidirectional carbon fiber/epoxy by reducing the shear tangent modulus of the matrix. The finite element analysis has shown that there are very high shear stresses near the ends of the tabs in both the ASTM and CRAG specimens, with peak values of about 75 and 90 MPa, respectively. There is also a significant longitudinal stress concentration at the end of the tabs for the straight tab CRAG specimen, but this effect is much less pronounced for the tapered tab ASTM specimen. It is therefore not surprising that failure usually initiates at the end of or just inside the tabbed section of the specimen for both of these configurations.

It is difficult to relate quantitatively the effect of the shear stresses found in the finite element analysis to the predicted reduction in compressive strength shown in Fig. 2 for a number of reasons. Both the longitudinal and shear stresses vary quite rapidly, whereas the Rosen model with reduced shear tangent modulus assumes constant stress. The finite element analysis also predicted shear stresses in the specimens larger than the maximum shear strength of the neat resin. This is due to inconsistency between the assumed composite shear behavior and the predicted behavior based on Rosen's model and the measured resin properties. There are a number of possible sources of discrepancies. The unidirectional shear properties were based on ±45-deg tension tests. This can result in errors due to material and geometric nonlinearity and the presence of other stress components apart from shear. Also there are discrepancies due to the expressions used to curve fit the data. The resin shear properties were derived from compression tests rather than shear tests. Different strain rates may have been used in the tests to measure composite and resin shear properties, and this may have had some effect. Another important factor is that Rosen's model is based on a two-dimensional representation of the composite whereas the behavior of the three-dimensional array of fibers in the matrix is considerably more complex. A further difficulty in relating the shear stresses to the predicted compressive strength is because of the constraining effect of the test fixture that is quite close to the point of failure initiation.

Despite these uncertainties, there is qualitative agreement. The modified Rosen equation would indicate that a shear stress of 45 MPa would reduce the compressive strength to about 1400 MPa. There are shear stresses of at least this magnitude over a significant portion of the specimen for both configurations. The shear stresses are somewhat lower in the tapered tab ASTM specimen, but this may be counterbalanced by the reduced support provided by the fixture due to the tapered tab. There may also be additional shear stresses as a result of misalignment between the fibers and loading axis.

This mechanism of reduced compressive strength due to shear stresses is absent in most of the tests where high compressive properties have been measured. In a laminate containing angle plies and loaded in compression, the 0-deg plies will tend to have very low shear stresses. Any shear that arises will be carried mainly by the angle plies. In a four point bending test shear stresses are very low in the central section and are zero at the surface where the compressive stress is a maximum and from where failure would be expected to initiate. With fiber tows embedded in epoxy with a very low volume fraction, the percentage of the total load carried by the fibers is small in contrast to the usual situation where the matrix carries negligible load. Shear stresses induced in the matrix in transferring load into the fibers will therefore be very small. In these cases the material can be expected to reach its full strength.

It would therefore appear that the ultimate compressive strength and strain to failure of unidirectional carbon fiber/epoxy are underestimated by a significant margin in indirect compression tests due to the presence of shear stresses. These arise primarily because of the method of loading but can also be caused by misalignment angles between the fibers and loading direction. The shear stresses reduce the shear tangent modulus of the matrix and hence the support that the matrix is able to provide to the fibers against buckling, leading to a reduced compressive strength.

The underestimation of unidirectional compressive strength could have important consequences from a design standpoint, given that it is often compression that controls the design of composite structures. It is unlikely that the true compressive strength could ever be attained in real unidirectional composite structures because there are likely to be misalignments within the material and shear stresses arising from the way loads are transferred into the structures. However, most applications of composites are for laminates with plies oriented in different directions. If low values of allowable layer compressive stress or strain are used in sizing laminates subject to compression, then an overconservative design will be produced. This can be seen in the results quoted earlier where experimental values of compressive stress and strain to failure for 0/90/±45-deg laminates were higher than predicted values by between 15 and 24%. More work is needed to investigate this and develop methods to enable accurate predictions of laminate compressive strength to be made.

V. Conclusions

Based on Rosen's classical theory of compressive strength for unidirectional composites, a large decrease in strength can be expected due to the presence of shear stresses. This is because of the reduction in tangent shear modulus caused by the nonlinear matrix behavior. High shear stresses arise in the ASTM and CRAG indirect compression test specimens commonly used to measure compressive strength. Shear stresses can also arise due to small misalignments between the fibers and the loading axis. It is therefore suggested that the intrinsic compressive strength of unidirectional carbon fiber/epoxy is underestimated by a significant margin by these tests. This conclusion is supported by the higher values of compressive strain achieved in laminated plates, bending tests, and single bundles of fibers embedded in epoxy in which significant shear stresses do not arise. The use of low values of unidirectional compressive strength derived from indirect compression tests in the design of composite structures using laminated plate theory could lead to significantly overconservative designs.

Appendix A: Shear Modulus Data for 914 Resin

The expressions for shear secant modulus in MPa in terms of shear strain γ from Ref. 16 are

$$G = \frac{1}{|\gamma|}\left\{-57 + \left[11990 - 10^6\left(0.0935 - |\gamma|\right)^2\right]^{1/2}\right\}$$

$$\text{for} \quad 0 \le |\gamma| \le 0.038$$

$$G = \frac{1}{|\gamma|}\left\{-113.5 + \left[33310 - 10^6\left(0.1405 - |\gamma|\right)^2\right]^{1/2}\right\}$$

$$\text{for} \quad 0.038 < |\gamma| \le 0.13$$

$$G = 51.52 + \frac{62}{|\gamma|} \qquad \text{for } |\gamma| > 0.13$$

These expressions can be multiplied by γ and differentiated to give equations for tangent shear modulus G_T. To avoid discontinuities in G_T present in the preceding equations, the ranges of γ for the three expressions have been modified.

$$G_T = 10^3\left(0.0935 - |\gamma|\right)\left[0.01199 - \left(0.0935 - |\gamma|\right)^2\right]^{-1/2}$$

$$\text{for} \quad 0 \le |\gamma| \le 0.023$$

$$G_T = 10^3\left(0.1405 - |\gamma|\right)\left[0.03331 - \left(0.1405 - |\gamma|\right)^2\right]^{-1/2}$$

$$\text{for} \quad 0.023 < |\gamma| \le 0.1311$$

$$G_T = 51.52 \qquad \text{for } |\gamma| > 0.1311$$

Appendix B: Material Properties Used in Finite Element Analysis

1) Unidirectional 60% volume fraction carbon fiber/epoxy properties from Ref. 16 (high strength surface treated fibers in 914 resin):

Longitudinal modulus = 131.69 GPa
Through thickness modulus = 9 GPa
Poisson's ratio = 0.335
Shear secant modulus G in terms of shear strain γ given by expressions

$$G = 4990 \qquad \text{for} \qquad |\gamma| < 0.00423672$$

$$G = 0.130324/|\gamma| + 5829.71 - 211715|\gamma| + 5.21158 \times 10^6|\gamma|^2 - 7.44307 \times 10^7|\gamma|^3 + 4.40223 \times 10^8|\gamma|^4$$

$$\text{for} \qquad 0.00423672 \le |\gamma| \le 0.055$$

$$G = 87.61/|\gamma| \qquad \text{for} \qquad |\gamma| > 0.055$$

The last expression is an extrapolation from the data in Ref. 16 assuming a constant shear stress at high shear strain.

Tangent modulus values were also required for the nonlinear solution. These were also taken from Ref. 16 for strains up to 0.055. To be consistent with the secant modulus for higher strains, a tangent modulus of 0 should have been used for strains greater than 0.055. This was avoided because of potential numerical problems.

$$G_T = 4990 \qquad \text{for} \qquad |\gamma| < 0.00214638$$

$$G_T = 5829.71 - 4.2343 \times 10^5|\gamma| + 1.56347 \times 10^7|\gamma|^2 - 2.977228 \times 10^8|\gamma|^3 + 2.201115 \times 10^9|\gamma|^4$$

$$\text{for} \qquad 0.00214638 \le |\gamma| \le 0.055$$

$$G_T = 1 \qquad \text{for} \qquad |\gamma| > 0.055$$

2) Cross-plied glass fiber/epoxy: values used are typical ones for woven cross-plied E-glass/epoxy of about 50% volume fraction. Shear stress-strain data was not readily available; however, since the shear modulus is very similar to carbon/epoxy, the same data was used as just presented. This was considered acceptable since the shear modulus is largely a matrix dominated property and the tab behavior was not of prime importance in the analysis.

Longitudinal modulus = 21.7 GPa
Through thickness modulus = 15.4 GPa
Poission's ratio = 0.28
Shear modulus as for carbon fiber/epoxy preceding

Acknowledgments

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